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MATHEMATICAL MODELING OF INFLATION PROCESSES IN THE ECONOMY USING DIFFERENTIAL EQUATIONS WITH FRACTIONAL DERIVATIVES

Background. ***When studying rapidly changing inflationary processes in economics, theoretical methods based on ordinary differential equations or partial differential equations are often used. However, as demonstrated in this paper, in certain cases it is more appropriate to use the apparatus of differential equations with fractional derivatives. This is due to the presence of various types of nonlinearities in functional relationships within inflationary processes, the influence of parameter values from previous time points on current values, the existence of scaling relations, and so on. In fact, all these characteristics are inherent to fractional calculus.***

Methods. ***The article is devoted to the application of differential equations with fractional derivatives of Caputo for the analysis of inflationary (deflationary) processes in the economy, based on the method of measuring inflation using the consumer price index, which takes into account changes in prices for a certain set of goods and services. This is demonstrated by changes in the specified index over finite time periods.***

Results. ***It is shown that the use of fractional order differential equations can be useful for building flexible tools for forecasting inflation/deflation processes. The relationship between the inflation rate and the unemployment rate is also investigated.***

Conclusions. ***It has been established that the change in the fractional derivative index in theoretical models of economic processes allows describing different regimes of price dynamics – from moderate inflation to galloping and hyperinflation, as well as complex deflationary scenarios. The appearance of negative values of price indices for individual goods can be interpreted as a consequence of their excess production, which leads to a loss of market value. The proposed method of using differential equations with fractional values of the order of derivatives provides an expansion of the possibilities of modeling a wide range of economic processes.***

Keywords: ***inflationary (deflationary) processes in the economy, price index, rate of price change, unemployment rate, Caputo's fractional derivatives, and differential equations with these derivatives.***

Background

In the modern economic environment, one of the key problems is inflation (Bossone, 2019; Dinh, 2020; Conrad, 2022; Lester, 2023; Afrouzi, 2024; Dibyendu, & Chandra, 2025; Ferraris, 2025). This phenomenon reflects the growth of the general level of prices for goods and services, which has a significant impact on the stability of financial systems and the socio-economic development of countries as a whole. The growth of prices and its consequences, such as the decline in the standard of living of the population and the deterioration of the macroeconomic situation, become the object of in-depth analysis and the search for optimal management strategies.

Researchers and practitioners draw attention to the importance of developing new methods and analytical tools for analyzing and forecasting inflationary phenomena. The relationship between inflation (deflation) and unemployment, and other economic indicators, is becoming a relevant task for economic science and practice.

The paper aims to achieve a more realistic and adaptive modeling of various types of inflationary processes using differential equations with fractional Caputo derivatives.

Literature review. The complex processes of inflation require the construction of various mathematical models for a deeper understanding. These models make it possible to predict such processes and to influence them through their interconnections with various economic and political decisions. For instance, in (Moza, Brandibur, & Găină, 2023), the relationship between interest rates, investment demand, and the inflation rate is studied using a four-dimensional model that describes these interactions by applying a control law to the interest rate. In (Ifeacho, & González-Parra, 2025), a mathematical model is proposed based on a system of first-order nonlinear differential equations, developed to study the impact of corruption, unemployment, and inflation on economic growth (it also considers numerical simulations where periodic solutions arise due to Hopf bifurcation). In works (Tsoularis, 2021; Navarro, & Tomé, 2022), economic models are also analyzed using ordinary differential equations, often allowing for single-factor approximations.

However, when modeling multifactor processes (for example, in the Black-Scholes model for option pricing), describing the dynamics of economic processes and systems (for example, in the analysis of inflationary processes in the economy), the possibility of describing optimal solutions, etc., partial differential equations provide greater flexibility, multidimensionality, and accuracy of approaches to mathematical modeling (see Alam, 2020; Neumann, 2022; Ashish, 2025; Kubba, & Abdou, 2025).

In particular, in recent years, differential equations with fractional derivatives have been increasingly used in economics (see, for example, (Dąbrowski, Janus, & Mucha, 2025; Saeidi, Hejazi, & Mohammadi, 2024; Tarasova, & Tarasov, 2017; Luo, Wang, & Fečkan, 2018; Ming, Wang, & Fečkan, 2019; Tarasov, 2020a, 2020b; Awa, 2020; Muhamad et al., 2021; Badi′k, & Fečkan, 2021; Cheow et al., 2024; Kocapor, Valério, & Radonjić, 2025)). Unlike ordinary differential equations and partial differential equations, fractional equations allow for modeling processes that take memory effects into account (for instance, in financial markets, prices are influenced not only by current news but also by previous trends and expectations). Fractional differential equations provide a smoother transition between linear and nonlinear growth, between stability and instability, which is important for modeling complex economic systems with many interdependent factors. Models with fractional derivatives tend to align better with empirical data than those based on conventional derivatives. Fractional derivatives also account for anomalous diffusion and time delays, which are crucial for adapting to economic situations with nonstandard dynamics – such as crises or long-term cycles. Since the economy encompasses various time scales – short-term fluctuations, medium-term cycles, and long-term trends – it makes sense to use fractional differential equations, as they can simultaneously account for the influence of all these scales. It should be noted that the order of a fractional derivative can be an arbitrary real number (and even an arbitrary complex number or a complex-valued function of a complex variable), which provides greater flexibility in tailoring models to specific problems. In this work, it will be shown that by varying the order of a Caputo-type fractional derivative, it is possible to describe different types of inflation in a unified manner. Finally, fractional equations naturally reflect so-called scaling properties, as they exhibit a self-similar (fractal) structure. Solutions to fractional equations often involve functions of the Mittag-Leffler type, which are characteristic of power-law distributions (for example, income distributions, company sizes, and price fluctuations).

In the works (Luo, Wang, & Fečkan, 2018; Ming, Wang, & Fečkan, 2019; Tarasov, 2020a, 2020b; Kocapor, Valério, & Radonjić, 2025), when analyzing statistical data on economic growth, respectively, in Spain, China, and Serbia, it was shown that the use of the fractional Caputo derivative leads to better results than when using derivatives of integer orders. It is obvious that using differential equations with fractional derivatives (in particular, the fractional Caputo derivative), it is possible to model inflationary (deflationary) processes, which we will briefly recall here.

Methods

A brief description of different types of inflation. Inflation can be given a fairly concise and extremely capacious definition: inflation – an increase in the general (average) price level over time (Bossone, 2019; Dinh, 2020; Conrad, 2022; Lester, 2023; Afrouzi, 2024; Dibyendu, & Chandra, 2025; Ferraris, 2025).

According to the rate of price growth, there are three types of inflation: moderate, galloping, and hyperinflation.

1) Moderate, or creeping, inflation occurs when prices in the country grow by an average of up to 10% per year. This type of inflation is considered safe, and when prices increase by only a few percent, it is even desirable. A slight increase in prices has a stimulating effect on economic entities, so to speak, "whips up" their business activity. The activation of demand, accordingly, stimulates production.

2) Galloping inflation occurs when prices increase by more than 10% per year (approximately 100 – 200%). This type of inflation becomes dangerous because when prices jump, people lose their composure, each time expecting another price increase.

3) Hyperinflation (from the Greek *hyper* – above) – the most dangerous type of inflation. During hyperinflation, prices grow extremely quickly; they seem to explode, reaching astronomical heights, for example, more than 1000% per year, or 50% per month, or 1% per day. This is self-accelerating inflation.

It should also be noted that with inflationary processes of price increases, there is a tendency to decrease prices (general or for individual types of goods). This process is called deflation. Deflation is caused by a shortage of money compared to the production of goods. This leads to a decrease in the rate of inflation – disinflation.

It is important to characterize the dynamics of the considered inflationary processes, both in discrete and continuous time, using the main quantitative characteristics (price indices for various goods, rates of change of these prices, the general core inflation index, the general inflation rate, etc.), to which we proceed in detail.

Formulation of the Cauchy problem for the consumer price index. One of the main indicators of inflation is the consumer price index (CPI), which reflects changes in prices for a certain set of goods and services (Ferraris, 2025; Vilenskyi, Lyvshits, & Smolyak, 2011). Based on this indicator, this article proposes a method for studying inflation processes using fractional order differential equations. However, before proceeding to the direct description of this method, it is demonstrated here how it is possible to first obtain an ordinary and fairly simple partial differential equation based on the analysis of the price index, and then make the transition to a fractional derivative differential equation.

The price index $J_k(t,s)$ for a good *k* for the period from time *s* to time *t* is the ratio of the price $P_k(t)$ at time *t* to the price $P_k(s)$ at time *s*:

$$J_k(t,s) = P_k(t)/P_k(s) . \tag{1}$$

In the case when the time point *s* is taken as the initial time point $t = 0$, the corresponding price index is called the base price index. Two main properties of base indices follow from the definition:

1) Reversibility: for any time instants *t* and *s* the equality holds:

$$J_k(t,s) = 1/J_k(t,s) . \tag{2}$$

Obviously, as follows from (2), for any *t* the equality holds $J_k(t,t) = 1$.

2) Transitivity: if $t_1, t_2, ..., t_m$ are arbitrary moments of time, then the following relation holds:

$$J_k(t_m,t_1) = J_k(t_2,t_1) \cdot J_k(t_3,t_2) \cdot ... \cdot J_k(t_m,t_{m-1}) . \tag{3}$$

Let us now turn to the consideration of the rate of change of the price $i_k(t)$ of product *k* at time *t*. The rate of change of the price of product *k* for the period from time *t* to time $t+\Delta$ is called the quantity

$$i_k(t+\Delta,t) = \left[P_k(t+\Delta) - P_k(t)/P_k(t)\Delta\right] . \tag{4}$$

Dividing the numerator and denominator of the right-hand side of expression (4) by $P_k(s)$, taking into account (1), w e obtain that for a given base time *s*, the value of the rate of change in the price of product *k* for the period from time *t* to time can be written in the following form:

$$i_k(t+\Delta,s)=\left[J_k(t+\Delta,s)-(t,s)\right]/J_k(t,s)\Delta\,. \quad (4a)$$

Expression (4a) is convenient because the price indices in it are reduced to a common base time point *s*. To obtain the rate of change of the price of product *k* at time point *t*, in (4a) we perform a limit transition by $\Delta\to 0$:

$$i_k(t,s)=\lim_{\Delta\to 0}\left[J_k(t+\Delta,s)-J_k(t,s)\right]/J_k(t,s)\Delta=$$

$$=\frac{1}{J_k(t,s)}\frac{\partial J_k(t,s)}{\partial t}=\frac{\partial}{\partial t}\ln\partial J_k(t,s). \quad (5)$$

As follows from formula (1), the right-hand sides of formulas (4a) and (5) do not really depend on the base time *s*. The main question is under what conditions the rate of price change $i_k(t,s)$ will depend only on the current time *t* and will not depend on the base time *s*. The answer is given by the following statement.

If for any step *m* the price index (hereinafter also the inflation index) $J_k(t,s)$ satisfies the transitivity condition (3), then the quantity $i_k(t,s)$ does not depend on *s*. Then the correct notation is $i_k(t)$. And, conversely, if $i_k(t,s)$ does not depend on *s* and it is known that for $J_k(t,s)$ (not necessarily given by formula (1)) the condition $J_k(t,t)=1$ is fulfilled, then satisfies $J_k(t,s)$ the transitivity condition (3) (for a proof of these statements by induction (Vilenskyi, Lyvshits, & Smolyak, 2011)).

Thus, for a price index defined in accordance with formula (1) (or for any other inflation indices that satisfy condition (3)), the rate of price change, as well as the inflation rate, is calculated by the formula

$$i_k(t)=\frac{1}{J_k(t,s)}\frac{\partial J_k(t,s)}{\partial t}. \quad (6)$$

The dimension of the rate of price change is 1/unit of time or %/unit of time (for example, % per year or % per month). Equation (6) is rewritten as follows:

$$\partial J_k(t,0)/\partial t=i_k(t)J_k(t,0)\,. \quad (7)$$

Being interested in the change in the price index $J_k(t,0)$ of the *k*-th product over a finite time interval $[0,T]$, we introduce the relative time $\tau=t/T$. As a result, equation (7) is rewritten as

$$\partial J_k(\tau)/\partial\tau=i_k(\tau)TJ_k(\tau)\,, \quad (8)$$

where $J_k(\tau,0)\equiv J_k(\tau)$. Assuming that the price of some *k*-th good during inflation can both increase and decrease, equation (8) is generalized to the following:

$$\partial J_k^{(\pm)}(\tau)/\partial\tau=\pm i_k(\tau)TJ_k^{(\pm)}(\tau)\,. \quad (8a)$$

Adding to it the initial condition $J_k^{(\pm)}(0)=1$, we proceed to the following Cauchy problem:

$$\partial J_k^{(\pm)}(\tau)/\partial\tau=\pm\lambda_k(\tau)J_k^{(\pm)}(\tau)\,,\ J_k^{(\pm)}(0)=1\,, \quad (9)$$

where $\lambda_k(\tau)=i_k(\tau)T$ is the dimensionless rate of change in the price of the *k*-th product over the time interval $[0,T]$. Solving problem (9) using the separation of variables method, we obtain the exponential growth (decrease) of the price index $J_k^{(\pm)}(\tau)$ depending on the relative time τ:

$$J_k^{(\pm)}(\tau)=\exp\left[\pm\int\lambda_k(\tau)d\tau\right]. \quad (10)$$

It is obvious that at $\lambda_k(\tau)=0$ from (10) we obtain $J_k^{(\pm)}(\tau)=1$, i.e. we have the case of a constant price of the *k*-th good (no inflation). If $\lambda_k(\tau)=\text{const}\equiv i_kT=\lambda_k$, then from (10) we obtain

$$J_k^{(\pm)}(\tau)=\exp(\pm\lambda_k\tau)\,, \quad (11)$$

that is, we have an exponential nature of the increase (decrease) in the price of the *k*-th good. Other functions $\lambda_k(\tau)$ can be taken if the rate of change itself changes in the process of inflation. However, there is another approach to the analysis of inflationary processes, which is based on the use of fractional derivatives (Caputo, & Mainardi, 1971; Samko, Kilbas, & Marichev, 1993; Kilbas, Srivastava, & Trujillo, 2006). In particular, in the future we will analyze various inflationary processes using differential equations with fractional derivatives (Caputo, & Mainardi, 1971; Kilbas, Srivastava, & Trujillo, 2006).

So, using the definition and properties of the consumer price index as one of the main indicators of the inflation process, and the justification of partial differential equations (8), (8a), and their solutions (10), (11), we proceed to the following generalization.

Consideration of inflationary processes using differential equations with fractional derivatives. Assuming that $\lambda_k(\tau)=\lambda_k$, from equations (8) and (8a) we proceed to the following differential equations with fractional derivatives:

$$\partial^\alpha J_k(\tau)/\partial\tau^\alpha=\lambda_kJ_k(\tau)\,, \quad (12)$$

$$\partial^\alpha J_k^{(\pm)}(\tau)/\partial\tau^\alpha=\pm\lambda_kJ_k^{(\pm)}(\tau)\,, \quad (12a)$$

where $n-1<\alpha<n,\ n=1,2,...$ (note that by changing the exponent of the fractional derivative α, as will be shown below, we can describe different inflation rates). Note also that as a fractional derivative $\partial^\alpha/\partial\tau^\alpha$ we will consider the left-hand fractional derivative of Caputo (Caputo, & Mainardi, 1971; Kilbas, Srivastava, & Trujillo, 2006):

$${}^CD_a^\alpha f(\tau)=\frac{1}{\Gamma(n-\alpha)}\int_a^\tau(\tau-t)^{n-\alpha-1}f^{(n)}(t)dt=$$

$$={}^RD_a^\alpha f(\tau)-\sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{\Gamma(k-\alpha+1)}(\tau-a)^{k-\alpha}\,, \quad (13)$$

where is the fractional Riemann – Liouville derivative (Kilbas, Srivastava, & Trujillo, 2006).

Results

Therefore, using (13) at $a=0$ equation (12a) is rewritten as follows:

$${}^CD_0^\alpha J_k^{(\pm)}(\tau)\mp\lambda_kJ_k^{(\pm)}(\tau)=0\,,$$

$$(\tau>0;n-1<\alpha<n;n\in\mathbb{N};\lambda_k\in\mathbb{R})\,. \quad (14)$$

Let's move on to analyzing the solutions of this fractional differential equation. In the case $0<\alpha<1$ the solution to equation (14) under the initial condition, as in equation (9), is the following function (Kilbas, Srivastava, & Trujillo, 2006):

$$J_k^{(\pm)}(\tau)=E_{\alpha,1}(\pm\lambda_k\tau^\alpha)\,, \quad (15)$$

where $E_{\alpha,\beta}(x)=\sum_{k=0}^{\infty}\left[x^k/\Gamma(\alpha k+\beta)\right]$ is the Mittag – Leffler function (Samko, Kilbas, & Marichev, 1993; Kilbas, Srivastava, & Trujillo, 2006). Appendix **A** discusses in detail the algorithm for solving equations of type (14). It is obvious that at $\alpha = 1$ the solution (15) will coincide with the function (11). Such functions of price indices for products with different inflation (deflation) rates are convenient for describing the processes of moderate (creeping) inflation (deflation).

In Fig. 1*a*, *b*, respectively, on finite time intervals $0 \le \tau \le 1$ and $0 \le \tau \le 10$ plots of the price index function are plotted, respectively, for the following values of the dimensionless rate of change in the price of the *k*-th product and the indices 〈 of the fractional derivative: $\lambda_k = 0, \pm 0.1T$, $\alpha = 1, 0.9, 0.7, 0.5$

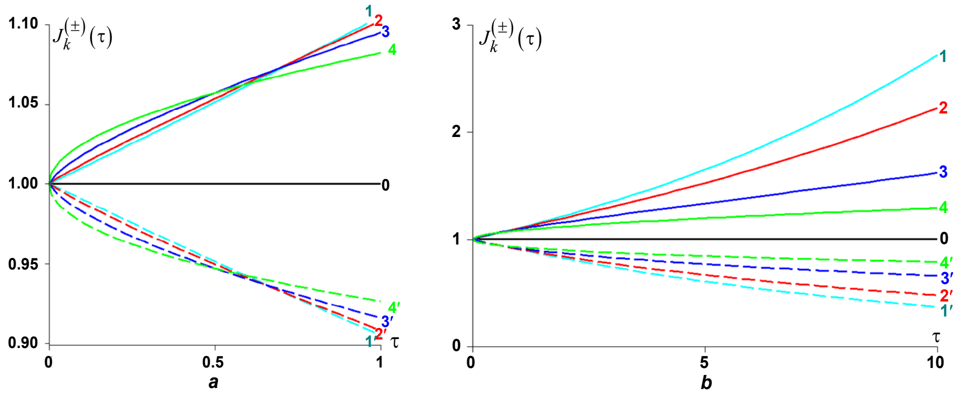

Fig. 1. Plots of the price index functions $J_k^{(\pm)}(\tau)$ of the *k*-th product at $0 \le \tau \le 1$ *a*), $0 \le \tau \le 10$ (*b*) for the cases $\lambda_k = 0$ – line 0; $\alpha = 1$ – curves 1, 1′; $\alpha = 0.9$ – curves 2, 2′; $\alpha = 0.7$ – curves 3, 3′, $\alpha = 0.5$ – curves 4, 4′

From Fig. 1*a* it is seen that for curves with smaller values of the indicator α to some values of times τ price indices will be higher than for curves with larger values of α. Then an inverse situation arises, when with lower indicators α there will be lower price indicators. As can be seen from Fig. 1*b*, this trend does not change even with further growth values of τ. It is obvious that here the nonlinear behavior of price indices (for example, for a specific product *k*) with changes in the inflation rate is manifested. For deflationary processes, as can be seen from Fig. 1*a*, *b*, the situation will be almost mirror-image for small values of τ and is strongly disturbed with increasing values of τ.

In the case $1 < \alpha < 2$ under initial conditions $J_k^{(\pm)}(0) = 1$ and $\partial J_k^{(\pm)}(0) / \partial \tau = \pm \lambda_k$ the solution to equation (14) is written in the following form (Kilbas, Srivastava, & Trujillo, 2006):

$$J_k^{(\pm)}(\tau) = E_{\alpha,1}\left(\pm \lambda_k \tau^{\alpha}\right) \pm \lambda_k \tau E_{\alpha,2}\left(\pm \lambda_k \tau^{\alpha}\right). \qquad (16)$$

In Fig. 2*a*, plots of price index functions $J_k^{(\pm)}(\tau)$ are plotted over a time interval $0 \le \tau \le 1$ for the same value λ_k and for the values of $\alpha = 1, 1.3, 1.5, 1.7$. In Fig. 2*b* plots of the price index function $J_k^{(\pm)}(\tau)$ for the same λ_k and α are plotted over a time interval $0 \le \tau \le 10$ (these functions $J_k^{(-)}(\tau)$ are examined separately below).

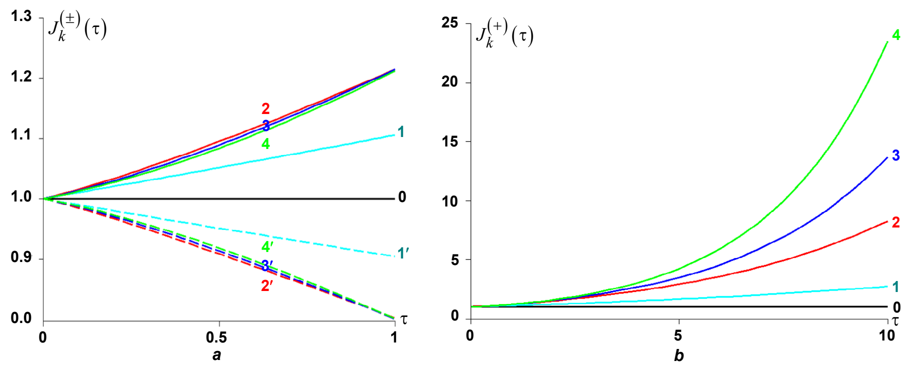

Fig. 2. Plots of the price index functions $J_k^{(\pm)}(\tau)$ of the *k*-th product at $0 \le \tau \le 1$ (*a*), $0 \le \tau \le 10$ (*b*) for the cases $\lambda_k = 0$ – line 0; $\alpha = 1$ – curves 1, 1′; $\alpha = 1.3$ – curves 2, 2′; $\alpha = 1.5$ – curves 3, 3′, $\alpha = 1.7$ – curves 4, 4′

As can be seen from Fig. 2*a*, the curves $J_k^{(+)}(\tau)$ with $1 < \alpha < 2$ pass above the curve with $\alpha = 1$ (in the case of inflation), and the curves $J_k^{(-)}(\tau)$ with $1 < \alpha < 2$ pass below the curve with $\alpha = 1$ (in the case of deflation). In this case, the behavior of the curves is similar to the behavior of the curves in Fig. 1*a*, namely, the curves with smaller values of α are initially located above the curves with larger values of α, and then they change places, which is a manifestation of the non-linearity of the functions (16). Note that using the function $J_k^{(+)}(\tau)$ processes with galloping inflation can be described (see Fig. 2*b*), and using the function $J_k^{(-)}(\tau)$ processes with galloping deflation can be described.

To describe hyperinflation we use the solutions of equation (14) at $2<\alpha<3$ and under initial conditions $J_k^{(+)}(0)=1$, $\partial J_k^{(+)}(0)/\partial\tau=\lambda_k$ and $\partial^2 J_k^{(+)}(0)/\partial\tau^2=\lambda_k^2$ (Kilbas, Srivastava, & Trujillo, 2006):

$$J_k^{(+)}(\tau)=E_{\alpha,1}\left(\lambda_k\tau^{\alpha}\right)+\lambda_k\tau E_{\alpha,2}\left(\lambda_k\tau^{\alpha}\right)+\lambda_k^2\tau^2 E_{\alpha,3}\left(\lambda_k\tau^{\alpha}\right). \quad (17)$$

Fig. 3 plots the function (17) for different values of the fractional derivative exponent (it is seen that with a change in the value of α in the interval, the function $J_k^{(+)}(\tau)$ increases sharply over time).

Finally, in Fig. 4*a*, *b* plots of the function $J_k^{(-)}(\tau)$ are plotted for those values of α that were chosen, respectively, when constructing the function $J_k^{(+)}(\tau)$ in Fig. 2*b*, Fig. 3 (in Fig. 4*a*, the time interval $0\le\tau\le 50$ was chosen, and in Fig. 4*b*, the time interval $0\le\tau\le 10$ was chosen, respectively). In addition, these figures show lines that occur when $\lambda_k=0$.

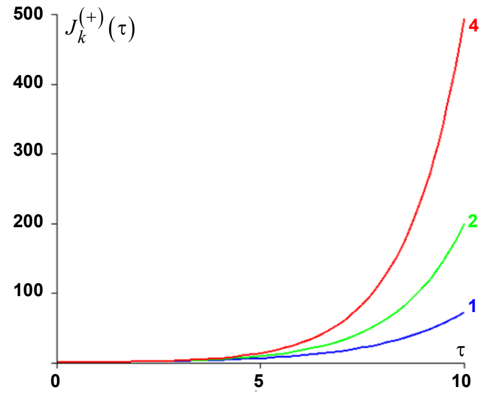

Fig. 3. Plots of the price index functions $J_k^{(+)}(\tau)$ of the *k*-th product over a time interval $0\le\tau\le 10$ for the cases $\alpha=2.1$ – curve 1, $\alpha=2.5$ – curve 2, $\alpha=2.9$ – curve 3

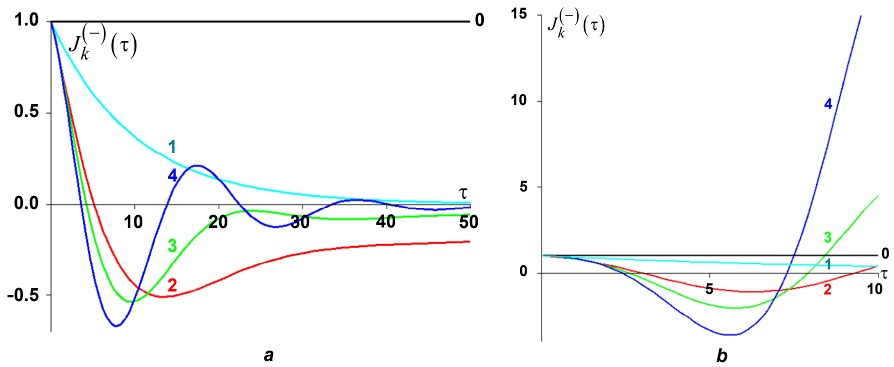

Fig. 4. Plots of the price index functions $J_k^{(-)}(\tau)$ of the *k*-th product: (*a*) – at $0\le\tau\le 50$ for $\alpha=1, 1.3, 1.5, 1.7$ (curves 1, 2, 3, 4); (*b*) – at $0\le\tau\le 10$ for $\alpha=1, 2.1, 2.5, 2.9$ (curves 1, 2, 3, 4); at $\lambda_k=0$ – line 0

Questions arise: how to understand a negative price index $J_k^{(-)}(\tau)$? Are such situations possible? Oddly enough, the answers to these questions can be positive. In this case, we can say that we are dealing with donations, charity, free assistance, etc.

From Fig. 4*a*, it can also be seen that in the case of galloping deflation, when moving from smaller values of α to larger ones, the change in the price index $J_k^{(-)}(\tau)$ acquires an increasingly pronounced damped oscillatory character. At the same time, this entire oscillatory process in the time interval $0\le\tau\le 50$ occurs at values $J_k^{(-)}(\tau)<1$ (curves 2 and 3, as can be seen from Fig. 4*a*, are mainly in the negative region).

Another trend, as can be seen from Fig. 4*b*, occurs in the case of hyperdeflation, when $2<\alpha<3$. In this case, when moving from smaller values of □ to larger ones, the price index $J_k^{(-)}(\tau)$ with the change in τ reaches an increasingly smaller negative value. Then as τ increases this index crosses both the abscissa axis and the dependence $J_k^{(-)}(\tau)=1$, and then it goes steeper and steeper towards infinity. This means that hyperdeflation transforms into hyperinflation.

Consideration of the relationship between inflation and unemployment over finite time periods using fractional differential equations. Let us now consider the relationship between inflation and the unemployment rate over finite time periods by analyzing solutions of differential equations with fractional derivatives for different values of the indicator α. As is known from the literature (Lester, 2023; Ferraris, 2025), in short-term periods, the relationship between the inflation rate *i* and the unemployment rate *b* is described by the Phillips curve. This dependence *i*(*b*) demonstrates that as the unemployment rate decreases, the inflation rate increases, i.e., by reducing unemployment, we "pay" with an increase in inflation. Note that the unemployment rate is calculated as the ratio of the number of unemployed people N_{up} registered with the state employment service to the number of working-age population N_{wp}, i.e. $b=N_{\text{up}}/N_{\text{wp}}$. We also note that the unemployment rate *b* should generally be much less than unity. However, theoretically, we can also consider situations where the parameter *b* can reach arbitrary positive values.

It is easy to see from the previous analysis that the simplest Cauchy problem describing such a relationship between inflation and the unemployment rate would be:

$$\frac{\partial i(b)}{\partial b} = -\mu i(b), \; i(0) = i_0, \qquad (18)$$

where the dimensionless quantity μ characterizes the rate of change in the inflation rate with an increase in the unemployment rate, i_0 is inflation in the absence of unemployment. Solving this equation, we obtain $i(b) = i_0 \exp(-\mu b)$. Equation (18) can be generalized by moving on to the following differential equation with fractional Caputo derivative:

$$D_0^{\alpha} i(b) + \mu i(b) = 0. \qquad (19)$$

Using expressions (15)–(17), the solutions of equation (19), for example, for the values $0 < \alpha_1 < 1$, $1 < \alpha_2 < 2$, $2 < \alpha_3 < 3$ are written in the following form:

$$i_1(b) = i_0 E_{\alpha_1,1}\left(-\mu b^{\alpha_1}\right). \qquad (20a)$$

$$i_2(b) = i_0\left[E_{\alpha_2,1}\left(-\mu b^{\alpha_2}\right) - \mu b E_{\alpha_2,2}\left(-\mu b^{\alpha_2}\right)\right]. \qquad (20b)$$

$$i_3(b) = i_0\left[E_{\alpha_3,1}\left(-\mu b^{\alpha_3}\right) - \mu b E_{\alpha_3,2}\left(-\mu b^{\alpha_3}\right) + \mu^2 b^2 E_{\alpha_3,3}\left(-\mu b^{\alpha_3}\right)\right]. \qquad (20c)$$

In Fig. 5 plots of functions (20a, b, c) are plotted, for example, for the following numerical values: $i_0 = 0.1$ in the time interval $[0, T]$, $\mu = 1$, $\alpha_1 = 0.5$, $\alpha_2 = 1.5$, $\alpha_3 = 2.5$.

As can be seen from Fig. 5, in addition to curve 1, which corresponds to the Phillips curve (Mankiv, 2000), other relationships between inflation and the unemployment rate are also possible. In particular, in the case of moderate inflation (curve 2), disinflation is higher for small values of *b* than for $\alpha = 1$, and, on the contrary, it is lower with increasing unemployment.

In the case of galloping inflation (for example, curve 3 in Fig. 5), the function *i*(*b*) at a certain unemployment level b_0 reaches a zero value and then, with the increase of *b* after reaching the minimum value, changes in a weakly oscillating manner in the negative region of its values. Here, an urgent question arises: what do negative values of the inflation rate mean? As above, when explaining a negative price index, negative values of the function *i*(*b*) occur in cases of charitable activities, free assistance, donations, etc.

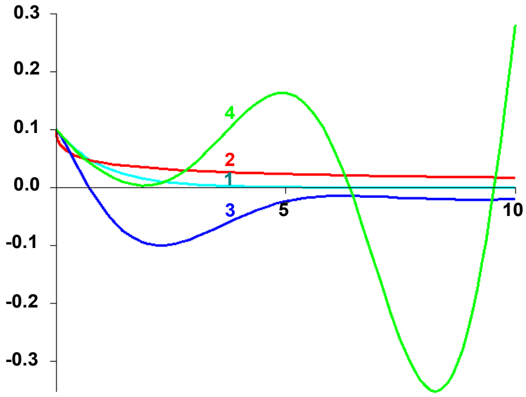

Fig. 5. Plots of the inflation rate function *i*(*b*) at $\mu = 1$ and for the following values of α : 1 – curve 1, 0.5 – curve 2, 1.5 – curve 3, 2.5 – curve 4

Finally, in the case of hyperinflation, the function *i*(*b*) (for example, curve 4 in Fig. 5) oscillates with increasing amplitude. Thus, as follows from this study, in addition to the traditional monotonic dependence (curve 1) between the unemployment rate and the inflation rate, mathematically admissible and significantly non-monotonic dependences (curves 2, 3, 4) are also significant. Obviously, detailed sociological studies are needed to confirm these non-trivial theoretical results.

It is also interesting to investigate the dependence of the inflation rate functions on the rate of change of the inflation rate for some specific values of unemployment levels. Thus, in Fig. 6a, b for the same values of α as in Fig. 5, graphs of the function $i(\mu)$ are plotted for the following unemployment levels: *b* = 0.1 (see Fig. 6*a*) and *b* = 0.5 (see Fig. 6*b*).

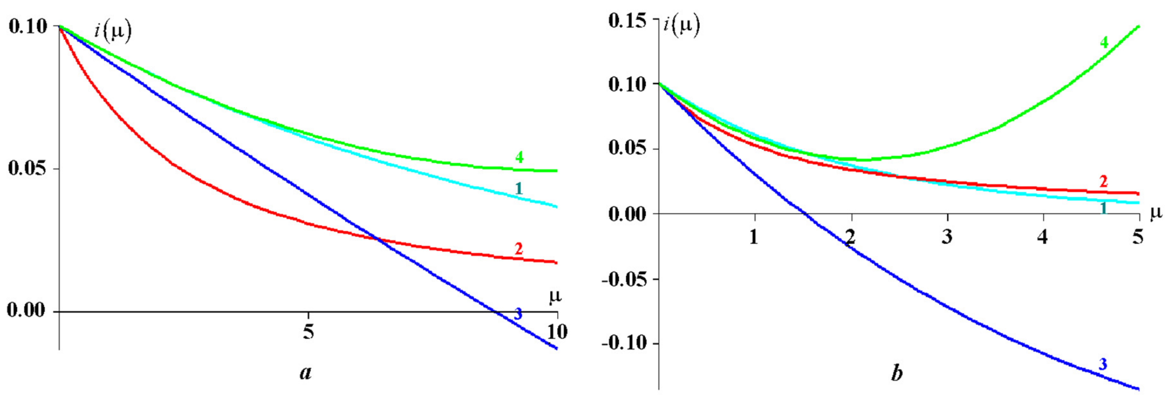

Fig. 6. Plots of the inflation rate function $i(\mu)$ for unemployment levels *b* = 0.1 – (*a*), *b* = 0.5 – (*b*) and for the following values of α : 1 – curves 1, 0.5 – curves 2, 1.5 – curves 3, 2.5 – curves 4

From Fig. 6*a*, *b* it is seen that the inflation rate function $i(\mu)$ at $\alpha > 1$ is characterized by strong non-monotonicity with increasing inflation rate change rate. It is seen that with increasing unemployment rate, minimum inflation rates are reached at lower inflation rate change rates. In the case of hyperinflation, these rates are the lowest.

Discussion and conclusions

Based on the use of differential equations with fractional derivatives (in particular, with Caputo derivatives), the work investigated changes in the price index, for example, of some *k*-th commodity (product) at different time intervals. It was shown that when the index α of the fractional derivative changes, using one fairly simple differential equation, it is possible to describe in a universal way changes in the price index under the influence of different rates of inflation processes, namely, under conditions of moderate, galloping, and hyperinflation. In addition, using this equation, the work also describes deflationary processes at different rates. At the same time, the non-trivial behavior of the obtained solutions may lead over time to the appearance of negative values of price indices (the latter can be explained by the presence of external influences).

Using the same differential equation, one can study the dependence of inflation rates on the unemployment rate at constant rates of change in inflation rates, as well as the dependence of inflation rates on the rates of change in inflation rates at constant inflation levels. It was shown that in addition to the classical Phillips dependence, which is realized at $\alpha = 1$, solutions of the corresponding differential equations with fractional exponents α can give radically different results, which open up prospects for more detailed studies.

Of course, an important point in the study of inflationary processes using differentials with fractional derivatives is the testing of the appropriate methods. Thus, using solutions of differential equations with fractional Caputo derivatives, it is possible to simulate changes in the prices of the main ingredients of the "borscht set" and changes in its average cost under conditions of moderate inflation. For this purpose, the possibility of continuously changing the parameters (in the region $1 < \alpha < 2$) and λ is used to reflect almost periodic changes in inflationary and deflationary periods against the background of general growth. It should be noted that similar dependencies are also found in the work (Lemishovskyi, & Dumych, 2024). It should be noted that for dependencies that describe hyperinflation (see, for example, the work (Tamimi, & Orbán, 2020), the parameters $2 < \alpha < 3$ and λ are easily selected.

Thus, the use of differential equations with fractional derivatives allows for more flexible modeling of complex inflation processes, which, as noted above, is also related to memory effects. In addition, analyzing on the basis of only one equation (with a change only in the order of the fractional derivative and initial conditions) simplifies the work.

The obtained mathematical relationships have the following interpretation:

1) the transition from lower to higher values of the order of the derivative corresponds to an acceleration of the inflation rate;

2) negative values of price indices in the solutions can be interpreted as a consequence of overproduction or changes in external conditions;

3) non-trivial relationships between inflation and unemployment demonstrate the possibility of both classical Phillips behavior and much more complex scenarios that require additional checks;

4) the model can describe the patterns of price fluctuations even under conditions of a variable inflation rate.

The practical significance of the results obtained lies in the possibility of:

1) building flexible tools for forecasting inflationary and deflationary processes;

2) operational monitoring of price indices (for example, for a set of basic food products) in different inflationary regimes;

3) adapting model parameters to specific economic conditions to take into account the influence of external factors (for example, the monetary policy of governments).

Finally, we note that fractional integro-differentiation is increasingly finding its application in various fields of natural science. In particular, in the study of the scattering processes of charged particles that are channeled in crystalline media (Maksyuta, Koshcheev, & Panina, 2012). Interesting results were also obtained when using fractional equations to consider the inflationary theory of the origin of our Universe (Rasoulia et al., 2023).

This is a consequence of the natural process of development/complication of the mathematical apparatus, which is required for a more detailed description of complex phenomena. And complex economic phenomena are no exception.

Appendix A

Using the left-sided fractional Caputo derivative (see formula (13)), we will find the solution to the differential equation

$${}^{C}D_0^{\alpha}y(t)-\lambda y(t)=0 \quad \left(t>0;\, n-1<\alpha\le n;\, n\in\mathbb{N};\, \lambda\in\mathbb{R}\right) \quad \text{(A1)}$$

using the following integral Laplace transform:using the following integral Laplace transform:

$$(\mathcal{L}y)(s)=\mathcal{L}\left[y(t)\right](s)=\int_0^\infty y(t)\exp(-st)\,dt\,. \quad \text{(A2)}$$

Using (A2), the Laplace transform of the fractional Riemann – Liouville derivative has the following form:

$$\mathcal{L}\left[{}^{R}D_{0+}^{\alpha}y(t)\right](s)=\int_0^\infty \exp(-st)\,D_{0+}^{\alpha}y(t)\,dt=$$

$$=\frac{1}{\Gamma(n-\alpha)}\int_0^\infty \exp(-st)\left[\frac{d^n}{dt^n}\int_0^t\frac{y(z)\,dz}{(t-z)^{\alpha-n+1}}\right]dt. \quad \text{(A3)}$$

After integrating by parts *n* times, expression (A3) will be written as follows:

$$\mathcal{L}\left[{}^{R}D_{0+}^{\alpha}y(t)\right](s)=\frac{s^n}{\Gamma(n-\alpha)}\int_0^\infty dt\int_0^t\frac{\exp(-st)\,y(z)\,dz}{(t-z)^{\alpha-n+1}}\,. \quad \text{(A4)}$$

Using the well-known Dirichlet formula

$$\int_a^b dx\int_a^x f(x,y)\,dy=\int_a^b dy\int_y^b f(x,y)\,dx$$

and moving to a new variable $w=t-z$, we continue with the following transformations:

$$\mathcal{L}\left[{}^{R}D_{0+}^{\alpha}y(t)\right](s)=\frac{s^n}{\Gamma(n-\alpha)}\int_0^\infty y(z)\,dz\int_z^\infty\frac{\exp(-st)\,dt}{(t-z)^{\alpha-n+1}}=$$

$$=\frac{s^n}{\Gamma(n-\alpha)}\int_0^\infty y(z)\,dz\int_0^\infty\frac{\exp\left[-s(z+w)\right]dw}{w^{\alpha-n+1}}=$$

$$=\frac{s^n}{\Gamma(n-\alpha)}\int_0^\infty y(z)\exp(-sz)\,dz\int_0^\infty \exp(-sw)\,w^{n-\alpha-1}dw=$$

$$=\left|\begin{matrix} sw=x,\ w=x/s \\ dw=dx/s\end{matrix}\right|=\frac{s^n}{\Gamma(n-\alpha)}\int_0^\infty y(z)\exp(-sz)\,dz\int_0^\infty \exp(-x)\,x^{n-\alpha-1}dx=$$

$$=s^\alpha\int_0^\infty y(z)\exp(-sz)\,dz=s^\alpha(\mathcal{L}y)(s)\,. \quad \text{(A5)}$$

We now find the Laplace transform of the second term in formula (13) under the condition $a=0$:

$$\mathcal{L}\left[\sum_{k=0}^{n-1}\frac{y^{(k)}(0)}{\Gamma(k-\alpha+1)}t^{k-\alpha}\right](s)=$$

$$\sum_{k=0}^{n-1}\frac{y^{(k)}(0)}{\Gamma(k-\alpha+1)}\int_0^\infty \exp(-st)\,t^{k-\alpha}dt=\left|\begin{matrix} st=x,\ t=x/s \\ dt=dx/s\end{matrix}\right|=$$

$$=\sum_{k=0}^{n-1}\frac{y^{(k)}(0)\,s^{\alpha-k-1}}{\Gamma(k-\alpha+1)}\int_0^\infty \exp(-x)\,x^{k-\alpha}dx=\sum_{k=0}^{n-1}y^{(k)}(0)\,s^{\alpha-k-1}\,. \quad \text{(A6)}$$

Therefore, subtracting expression (A6) from expression (A5), we arrive at the following Laplace transform of the fractional Caputo derivative

$$\mathcal{L}\left[{}^{C}D_{0+}^{\alpha}y(t)\right](s)=s^{\alpha}\mathcal{L}\left[y(t)\right](s)-\sum_{j=0}^{n-1}d_{j}s^{\alpha-j-1}, \quad (A7)$$

where $d_j = y^{(j)}(0)$, $(j=0,...,n-1)$. Now applying (A2) and (A7) to equation (A1), we obtain

$$\mathcal{L}\left[y(t)\right](s)=\sum_{j=0}^{n-1}d_{j}\frac{s^{\alpha-j-1}}{s^{\alpha}-\lambda}. \quad (A8)$$

On the other hand, it is easy to show that the following formula holds:

$$\mathcal{L}\left[t^{j}E_{\alpha,j+1}\left(\lambda t^{\alpha}\right)\right](s)=\frac{s^{\alpha-j-1}}{s^{\alpha}-\lambda}\quad\left(\left|s^{-\alpha}\lambda\right|<1\right). \quad (A9)$$

Let's prove this formula:

$$\mathcal{L}\left[y_{j}(t)\right](s)=\int_{0}^{\infty}t^{j}E_{\alpha,j+1}\left(\lambda t^{\alpha}\right)\exp(-st)\,dt=$$

$$=\sum_{k=0}^{\infty}\frac{\lambda^{k}}{\Gamma(\alpha k+j+1)}\int_{0}^{\infty}t^{\alpha k+j}\exp(-st)\,dt. \quad (A10)$$

Further performing a linear substitution $st = x$ in the integral of formula (A10), we obtain

$$\int_{0}^{\infty}t^{\alpha k+j}\exp(-st)\,dt=\left|\begin{matrix}x=st,\ t=x/s\\ dt=dx/s\end{matrix}\right|=$$

$$=\frac{1}{s^{\alpha k+j+1}}\int_{0}^{\infty}x^{\alpha k+j}\exp(-x)\,dx=\frac{\Gamma(\alpha k+j+1)}{s^{\alpha k+j+1}}. \quad (A11)$$

Substituting (A11) into (A10), we obtain

$$\mathcal{L}\left[y_{j}(t)\right](s)=\frac{1}{s^{j+1}}\sum_{k=0}^{\infty}\left(\lambda s^{-\alpha}\right)^{k}. \quad (A12)$$

From formula (A12) it is clear that the power series present in it is a geometric progression with denominator $q=\lambda s^{-\alpha}$. It is obvious that under the condition $\left|\lambda s^{-\alpha}\right|<1$ we have

$$\sum_{k=0}^{\infty}\left(\lambda s^{-\alpha}\right)^{k}=\frac{1}{1-\lambda s^{-\alpha}}=\frac{s^{\alpha}}{s^{\alpha}-\lambda}. \quad (A13)$$

So, after substituting (A13) into formula (A12) under the condition $\left|\lambda s^{-\alpha}\right|<1$ we arrive at formula (A9). Thus, from relations (A8) and (A9) it follows that the solution of equation (A1) is represented in the following form:

$$y(t)=\sum_{j=0}^{n-1}d_{j}y_{j}(t), \quad (A14)$$

where $y_j(t)=t^{j}E_{\alpha,j+1}\left(\lambda t^{\alpha}\right)$, $(j=0,...,n-1)$ is the fundamental system of solutions of equation (A1).

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ІНФЛЯЦІЙНИХ ПРОЦЕСІВ В ЕКОНОМІЦІ ЗА ДОПОМОГОЮ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ ІЗ ДРОБОВИМИ ПОХІДНИМИ

Вступ. ***Під час вивчення швидкозмінних інфляційних процесів в економіці часто використовують теоретичні методи, засновані на звичайних диференціальних рівняннях або диференціальних рівняннях із частинними похідними. Однак, як показано в цій статті, у певних випадках доцільніше застосовувати апарат диференціальних рівнянь із дробовими похідними. Це пов'язано з наявністю різних типів нелінійностей у функціональних зв'язках у межах інфляційних процесів, впливом значень параметрів із попередніх моментів часу на поточні значення, існуванням співвідношень масштабування тощо. Фактично, всі ці характеристики притаманні дробовому численню.***

Методи. ***Статтю присвячено застосуванню диференціальних рівнянь із дробовими похідними Капуто для аналізу інфляційних (дефляційних) процесів в економіці, базуючись на методі вимірювання інфляції з використанням індексу споживчих цін, який враховує зміни у цінах на певний набір товарів і послуг. Це продемонстровано на змінах зазначеного індексу на скінченних часових відрізках.***

Результати. ***Показано, що використання диференціальних рівнянь дробового порядку може бути корисним для побудови гнучких інструментів прогнозування процесів інфляції / дефляції. Також досліджено зв'язок між рівнем інфляції та рівнем безробіття.***

Висновки. ***Встановлено, що зміна індексу дробової похідної в теоретичних моделях економічних процесів дозволяє описувати різні режими цінової динаміки – від помірної інфляції до галопуючої та гіперінфляції, а також складні дефляційні сценарії. Поява від'ємних значень індексів цін на окремі товари може бути інтерпретована як наслідок їх надлишкового виробництва, що призводить до втрати ринкової вартості. Запропонований метод використання диференціальних рівнянь із дробовими значеннями порядку похідних забезпечує розширення можливостей моделювання широкого спектра економічних процесів.***

Ключові слова: ***інфляційні (дефляційні) процеси в економіці, індекс цін, темпи зміни цін, рівень безробіття, дробові похідні Капуто та диференціальні рівняння із цими похідними.***

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